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A TABLE OF THE MODIFIED BESSEL FUNCTIONS

$K_n(x)$ AND $I_n(x)$ TO AT LEAST 60S FOR $n = 0, 1$ AND $x = 1, 2, \dots, 40$

By

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Assisted By

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The most extensive high accuracy tables of $K_n(x)$ were computed by J. C. P. Miller and C. E. Gwyther to 20S for the ranges $n = 0, 1, 2, \dots, 50$ and $x = 1, 2, 3, \dots, 100$. [1]. The tables presented here supplement those of Miller and Gwyther by providing higher accuracy pivotol values for the computation of $K_n(x)$ in the middle range of the arguement.

Our computations were based on the ascending series

$$K_n(x) = \frac{1}{2} \left(\frac{1}{2} x\right)^{-n} \sum_{k=0}^{n-1} \frac{(n-k-1)!}{k!} \left(-\frac{1}{4} x^2\right)^k + (-1)^{n+1} \ln \left(\frac{1}{2} x\right) I_n(x) + (-1)^n \frac{1}{2} \left(\frac{1}{2} x\right)^n \sum_{k=0}^{\infty} \{\psi(k+1) + \psi(n+k+1)\} \frac{\left(\frac{1}{4} x^2\right)^k}{k!(n+k)!} \quad (1)$$

$$I_n(x) = \left(\frac{1}{2} x\right)^n \sum_{k=0}^{\infty} \frac{\left(\frac{1}{4} x^2\right)^k}{k! \Gamma(n+k+1)} \quad (2)$$

where

$$\psi(n) = -\gamma + \sum_{k=1}^{n-1} k^{-1}$$

Euler's constant γ and the natural logarithm were taken from [2].

Depending on the magnitude of the arguement, x , between 80 and 120 terms were retained in the series (1) and (2). All computations

were carried out retaining 100 digits utilizing multiple precision routines modified from [3]. A check was provided by the Wronskian

$$I_n(x) K_{n+1}(x) + I_{n+1}(x) K_n(x) = 1/x \quad (3)$$

The number of digits shown in the table for each value of the argument, x , corresponds to the number of digits to which the equality (3) was satisfied when the computed values for $I_n(x)$ and $K_n(x)$ were substituted. However it must be noted that (3) is satisfied for arbitrary values of Euler's constant and the natural logarithm. In order to reduce the possibility of error in these quantities, Euler's constant was checked from several sources and the natural logarithms were independently verified by the authors by comparison to the values given in [2]. Dr. John W. Wrench Jr. provided hand computed results of high accuracy for $x = 10$. These were in complete agreement with the values given here.

The computation of $K_n(x)$ in the middle range of values, $2 < x < 40$, is a tedious task because of the slow convergence of the ascending series and the low accuracy of the asymptotic expansions. This table provides pivotal values for computation of $K_0(x)$ and $K_1(x)$ for any x through the series, (4) and (5)

$$K_0(x-h) = K_0(x) I_0(h) + 2 \sum_{r=1}^{\infty} K_r(x) I_r(h) \quad (4)$$

$$K_1(x-h) = K_1(x) I_0(h) + \sum_{r=1}^{\infty} \{K_{r+1}(x) + K_{r-1}(x)\} I_r(h) \quad (5)$$

This method has been applied with good results by the author in [4].

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2. W. E. Mensell, Tables of Natural and Common Logarithms to 110 Decimals, Royal Society Mathematical Tables, Vol. 8, 1964.
3. G. Kacin, Multiple Precision Package, Share SDA 3471-3482.
4. B. S. Berger, "Dynamic Response of an Infinite Cylindrical Shell in an Acoustic Medium", Journal of Applied Mechanics, Vol. 36, Series E, No. 2, June 1969, pp. 342-345.

A TABLE OF MODIFIED BESSEL FUNCTIONS

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I0( 1) = ( -1 ) 0. 1266065877 7520083355 9824462521 4717537607 6703113549 6220680813 5331213575 0161227754 7039481835 71472801
I0( 2) = ( -1 ) 0. 2279585302 3360672674 3720444081 1533353285 8411027854 5905407083 9751664305 3432326763 4272951708 85564858
I0( 3) = ( -1 ) 0. 4880792585 8650240856 1123554602 1319249424 3288564059 9761605956 3816165892 5779685827 0373428963 2694045
I0( 4) = ( -2 ) 0. 1130192195 2136330496 3562701832 1710249741 2616594435 3377060064 9619347053 4465082183 6707491482 693062
I0( 5) = ( -2 ) 0. 2723987182 3604446894 5442320758 8441928247 9061632220 9838151716 5811040289 9840706629 2719784364 9715
I0( 6) = ( -2 ) 0. 6723440697 6477975326 1880251448 7659014539 3097699676 3122887475 7692860645 4585847157 5121627045 491
I0( 7) = ( -3 ) 0. 1685939085 1028969885 7326627187 5008403765 2267923453 1714193194 0556685541 6412467567 8265231691 81
I0( 8) = ( -3 ) 0. 4275641157 2180478517 7396791318 0828851255 6410479123 7115884297 2189022251 7446587810 9256192390 4
I0( 9) = ( -4 ) 0. 1093588354 5113746958 4582425142 5470842609 4639321617 3571510148 2776940898 6654384660 7194943405
I0(10) = ( -4 ) 0. 2815716628 4662544714 6981115342 6590093078 4512383960 7778216714 8259930573 9028116590 260073609
I0(11) = ( -4 ) 0. 7288489339 8212481061 8144396107 6268087674 2632676532 4387227705 1541626049 8551009816 964887171
I0(12) = ( -5 ) 0. 1894892534 9296308861 2081082297 5396610096 3950870204 7517844419 4212472379 4092665454 04300787
I0(13) = ( -5 ) 0. 4944448958 2217572999 2545976985 6286707468 6667334837 1619760370 0170040544 1569237798 7264759
I0(14) = ( -6 ) 0. 1294185627 0064856366 4588328477 9364697034 5066538686 8344002400 6502260200 6744063678 8373352
I0(15) = ( -6 ) 0. 3396493732 9791387952 1701629914 9152510183 5150329463 6276630247 1709286967 5741074445 88643
I0(16) = ( -6 ) 0. 8934462279 2010501707 0864030976 1884542780 6981885885 3298423718 9283087257 3859792236 0071
I0(17) = ( -7 ) 0. 2354970223 1682933731 9939233299 2929747499 6698553672 7914608766 2349473770 3337528618 3515
I0(18) = ( -7 ) 0. 6218412420 7810029498 6247775552 2804357608 5674310754 3107914088 5033408752 3836386209 685
I0(19) = ( -8 ) 0. 1644619044 0611710822 0206046187 3027324489 6277123252 2165999496 3911042355 0575616588 239
I0(20) = ( -8 ) 0. 4355828255 9553533272 1066600892 1769191706 7099482746 5269930168 2073787535 2474459323 0
I0(21) = ( -9 ) 0. 1155139619 2215806412 1005558083 3193182349 0083586833 2920462491 7574832164 8523172328 4
I0(22) = ( -9 ) 0. 3066929936 4036474561 3498915355 7154852040 4680189132 4858455872 8394421589 490260972
I0(23) = ( -9 ) 0. 8151421225 1289225651 7156135743 7946877416 8333456834 6507045933 3522318542 742157946
I0(24) = ( -10 ) 0. 2168519088 2413765036 8899361766 6420248551 0296142249 8367439658 0204950627 66736754
I0(25) = ( -10 ) 0. 5774560606 4663103157 7133979730 6938266433 2453978739 3670045504 9101148723 7281323
I0(26) = ( -11 ) 0. 1538897670 5660810462 8354178833 6136099658 2329792861 0316806121 8270253472 452627
I0(27) = ( -11 ) 0. 4104200585 4757955531 4032727239 0385136563 1320675415 7648253497 4361936144 78481

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I0(28) = (12) 0. 1095346047 3175727348 2674856440 1226073645 7320378007 2536561256 7578493112 948
I0(29) = (12) 0. 2925206317 8569086626 5711641029 3661481220 6762389396 1483658707 2399357815 5
I0(30) = (12) 0. 7816722978 2397748971 7389816705 2950054449 4425397794 7003347688 77258125
I0(31) = (13) 0. 2089962966 4919037954 3811289099 3681402263 4491103496 7623896543 70889
I0(32) = (13) 0. 5590908381 3508730865 0010377608 8605327594 3304276596 3325212013 9
I0(33) = (14) 0. 1496379690 4156905651 9181463847 3178109312 3329233220 2193255949 544658982
I0(34) = (14) 0. 4006857504 2540716033 6123864914 1784837548 3205750833 8232949892 070011228
I0(35) = (15) 0. 1073388184 9451406357 3489383846 4930548254 2105898700 6903814791 5286571
I0(36) = (15) 0. 2876667778 1124404291 0293901874 3956743168 5978115853 8325992014 3995998
I0(37) = (15) 0. 7712455222 9281050494 7038702878 6917387436 4484346341 8885469375 886349
I0(38) = (16) 0. 2068504714 8510448451 8831503026 5960321539 5027979501 6289133046 90352
I0(39) = (16) 0. 5549742955 3141837649 2838538633 4394109192 5439699816 5737206636 2891
I0(40) = (17) 0. 1489477479 3419899924 2245915707 2118444494 6659556670 9917645085 521

A TABLE OF MODIFIED BESSEL FUNCTIONS

I1(1) =	(0)	0.	5651591039	9248502720	7696027609	8633073288	9962162109	2009480294	4894792556	4096437113	4092664997	76681440
I1(2) =	(1)	0.	1590636854	6373290633	8225442499	9666247954	4781594955	3664713228	7984608545	0375353611	8511612214	75942289
I1(3) =	(1)	0.	3953370217	4026093964	7863574058	0581287584	2215950912	5917758584	5489515361	7968310238	4358303051	2645192
I1(4) =	(1)	0.	9759465153	7044499094	7519256731	2680900055	9703332529	6730692752	8051449699	9466063234	8623709555	586421
I1(5) =	(2)	0.	2433564214	2450527199	1430504517	6000846056	4874368298	8981584030	8023040396	2968716841	6786172004	1842
I1(6) =	(2)	0.	6134193677	7640237861	3293101145	8462953361	2328192810	3397373792	1721320452	2165112928	6394628792	257
I1(7) =	(3)	0.	1560390928	6995545346	2390580660	7111556300	3105204154	4943170624	8746148703	3616940013	3520036614	52
I1(8) =	(3)	0.	3998731367	8256009821	9083086145	8227548896	2844390406	7647306574	3717330047	2347142464	6726980577	1
I1(9) =	(4)	0.	1030914722	5169564444	2875486360	6359038756	7213667920	2269937268	0752813032	4551370811	9155492011	
I1(10) =	(4)	0.	2670988303	7012546543	4103196677	2152549145	7451537875	3771310848	9316194548	2750798663	396557450	
I1(11) =	(4)	0.	6948858659	8121632308	2096427740	3070724535	6713134259	5124291024	4272779507	7980717330	087340508	
I1(12) =	(5)	0.	1814134878	1638831601	4252147956	2888222329	5359608594	7218474368	8480874944	3254478964	05903417	
I1(13) =	(5)	0.	4750298735	8995860929	3595802623	3749200530	9074796158	4250179480	4375959255	3456050602	3469275	
I1(14) =	(6)	0.	1247072591	4906986035	0493462759	9926193931	5969003648	6607143147	9589927148	6892725353	0671506	
I1(15) =	(6)	0.	3281249219	7020639673	3698150245	9844079265	5583311133	3381215373	3811338567	4767535891	20973	
I1(16) =	(6)	0.	8650594358	5483947141	8076217495	2913872674	4748495488	2688667001	4900848396	0956253071	7729	
I1(17) =	(7)	0.	2284621583	8080798026	3887272582	4144373015	4728708565	1289760586	4373543251	5694830618	5143	
I1(18) =	(7)	0.	6043133242	1156283704	0703902352	1274191615	1933356670	4093671120	0850700430	4149485484	094	
I1(19) =	(8)	0.	1600737378	5836995498	3738115808	5445488029	4377275570	3429817467	5222497302	0483008594	378	
I1(20) =	(8)	0.	4245497338	5127770181	4099066585	5938402281	2091932829	7879278873	7793112956	4627201272	5	
I1(21) =	(9)	0.	1127291991	3777552577	2339019881	7116628405	9724515980	5557511220	3209913615	4736846959	7	
I1(22) =	(9)	0.	2996396068	7737897972	6336305096	3381156794	8952416385	1421222424	9947904593	810828154		
I1(23) =	(9)	0.	7972200260	8965031677	0993120851	5629685495	2412120215	4685108874	3715965425	700068274		
I1(24) =	(10)	0.	2122947893	2873137510	0177852923	3270010858	9196027585	6766239960	2195801123	97249600		
I1(25) =	(10)	0.	5657865129	8787013531	0388858727	0006860623	5164070949	2977694185	8238659792	0910252		
I1(26) =	(11)	0.	1509007264	2341644305	3071893268	3130000506	5476751573	4337592546	5527711234	092665		
I1(27) =	(11)	0.	4027465431	3438059792	6579302404	4846234206	3180306803	7642573242	1143870524	43817		

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I1(28) = ( 12) 0. 1075605042 0808224413 9940325989 7041769965 2550765036 0151036416 5728057771 515
I1(29) = ( 12) 0. 2874321081 2625481210 9094872869 4058475269 8416425202 2944311141 5207075540
I1(30) = ( 12) 0. 7685320389 3895699949 4294710788 1801819071 3878837379 9676725684 90502015
I1(31) = ( 13) 0. 2055972795 2945647312 2155230752 0250662870 0532615311 4123366316 84023
I1(32) = ( 13) 0. 5502845511 2112481869 2486056144 6296048318 7494358268 6493165610 9
I1(33) = ( 14) 0. 1473530041 9212181612 6743188903 4932625738 0896259247 7601016013 680933215
I1(34) = ( 14) 0. 3947486494 1248591312 1246953030 9781282105 4569600061 1013731639 627194647
I1(35) = ( 15) 0. 1057941260 5189626610 5006371936 3974954451 3415668933 1823986779 3253564
I1(36) = ( 15) 0. 2836428540 1134374184 2703340139 2555958788 8829209579 7469411734 4477265
I1(37) = ( 15) 0. 7607508764 9264155690 8497226010 8309190294 7752065138 8174259596 118459
I1(38) = ( 16) 0. 2041103569 9813687124 5152521025 3853360426 6905729640 2298919785 14057
I1(39) = ( 16) 0. 5478124121 3094898470 2684002723 9960055027 1657707614 6507098188 6389
I1(40) = ( 17) 0. 1470739616 3259352738 8169580534 8819289591 8183259222 8673967350 661
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A TABLE OF MODIFIED BESSEL FUNCTIONS

K0(1) =	(0)	0.	4210244382	4070833333	5627379212	6093613621	9748226660	4722989690	5514552126	7813810183	9092125141	56486141
K0(2) =	(0)	0.	1138938727	4953343565	2719574932	4818329983	2662438880	8882892529	7899776631	9520680631	1655975109	79582447
K0(3) =	(-1)	0.	3473950438	6279248072	3495513510	8883169484	5980619042	0292533206	0113450932	2290824825	0078432059	5217274
K0(4) =	(-1)	0.	1115967608	5853024269	7451959798	3348922500	9023888474	3405382552	6153728917	9214519507	1516205334	942695
K0(5) =	(-2)	0.	3691098334	0425942747	3526100745	6995099001	9443446957	2132630190	1770603642	5380392757	3744009228	8991
K0(6) =	(-2)	0.	1243994328	0131230852	3246926008	8441397095	6319070621	8953892807	6634947633	2523491356	7053965852	988
K0(7) =	(-3)	0.	4247957418	6923180685	1598652806	5722939793	1752463243	1513571579	4459822509	5002003175	1311290235	63
K0(8) =	(-3)	0.	1464707052	2281538709	6584408698	6779219673	0536883375	9024089153	9984315639	2894225896	3326031175	6
K0(9) =	(-4)	0.	5088131295	6459247569	7393666843	9972896390	4939541792	8157553738	9201371931	6557782617	5771060910	
K0(10) =	(-4)	0.	1778006231	6167651811	3011927994	9279231287	3470160346	4360092539	1839904456	6305689925	689864939	
K0(11) =	(-5)	0.	6243020547	6536771452	9067351791	7081138735	0143720173	1640353399	4562030446	5630905979	824800186	
K0(12) =	(-5)	0.	2200825397	3114914005	1559977454	5416690691	1072008626	6998529290	4146686684	2407603920	77942135	
K0(13) =	(-6)	0.	7784543861	4204963208	2183555236	9360549165	7583401293	4928788035	3557527098	2340413498	4329419	
K0(14) =	(-6)	0.	2761370823	9816198913	2366961666	8448338799	1237742332	9311608751	8240958820	7218521048	5052511	
K0(15) =	(-7)	0.	9819536482	3964345409	9136592788	4599856019	9277508107	6471291485	9356235599	3333091567	54480	
K0(16) =	(-7)	0.	3499411663	9364989360	4550453251	4608259428	2524643968	1548594647	5874083179	2429169172	9223	
K0(17) =	(-7)	0.	1249466402	6317731910	9592943065	3536190571	6493922746	6881273587	7154956584	8458808377	8333	
K0(18) =	(-8)	0.	4468753337	3093829196	6717957377	8947299000	0726117027	8603035429	8369972150	5067936205	432	
K0(19) =	(-8)	0.	1600671286	9293614528	5207061824	3366098391	5499233123	4620872361	9960663730	6190239131	904	
K0(20) =	(-9)	0.	5741237815	3365242927	1670206162	2973781364	2403637973	4925755870	1002239107	5404040180	8	
K0(21) =	(-9)	0.	2061767969	9853177299	3156034395	0529285409	5343204579	1037723278	5439379755	7518830674	6	
K0(22) =	(-10)	0.	7412351614	0848653689	4645729194	3861332421	9402447109	6989681625	2535083720	007843842		
K0(23) =	(-10)	0.	2667545110	3519100119	5170530544	8692617099	0893158437	5139236547	8845240097	810960319		
K0(24) =	(-11)	0.	9608818780	8331156932	6318990203	6930883698	2692263632	9261117361	7305131645	15752325		
K0(25) =	(-11)	0.	3464161562	2131143553	9853822297	4494324307	5492577057	7307064167	7618154468	5303227		
K0(26) =	(-11)	0.	1249877397	9850723773	3556221909	0037200228	3373241458	0842024834	2476581395	912898		
K0(27) =	(-12)	0.	4512364531	1911031808	7110119760	1854378460	4689774051	3315366017	8114163553	34306		

K0(28) = (-12) 0. 1630534586 8881810373 5696816891 5746334347 3800599816 5373181004 6539024719 664
K0(29) = (-13) 0. 5894950728 7925601035 4618504736 7411076653 6232533605 3332314951 3420511502 8
K0(30) = (-13) 0. 2132477496 4630563711 6689606296 5376439927 0618437288 8796319678 26287572
K0(31) = (-14) 0. 7718382655 5276153837 8121093048 7419574792 5383248637 5168902271 31802
K0(32) = (-14) 0. 2795057518 7619787705 7813459226 9409471089 1229506988 8837968061 7
K0(33) = (-14) 0. 1012661232 3859171337 6482166831 0558820301 7520024214 9086531432 813268579
K0(34) = (-15) 0. 3670575965 8565077653 1280974605 7927289685 2897940470 8240820572 025634156
K0(35) = (-15) 0. 1331035149 1429468528 3495547194 9796975827 7813222922 1203828376 1848101
K0(36) = (-16) 0. 4828583383 1109771295 4267517475 0153778884 3019425533 3840824810 9775394
K0(37) = (-16) 0. 1752327700 4675630268 4877514899 5763492491 5989039409 2452762344 394994
K0(38) = (-17) 0. 6361617158 8690491714 4806936360 9144129049 8654645244 2402128721 76997
K0(39) = (-17) 0. 2310299363 7829395687 1303735926 5927532387 1535190053 6451498225 8681
K0(40) = (-18) 0. 8392861100 0995670337 2830790498 9453021194 4757297082 6546842379 603

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K1(1) =	(0)	0.	6019072301	9723457473	7540001535	6173392615	8688996810	6456017761	9591685535	8294623784	0168863706	05766820
K1(2) =	(0)	0.	1398658818	1652242728	4598807035	4110238872	3458484151	5530384442	0543185617	6640539863	9293899713	67817175
K1(3) =	(-1)	0.	4015643112	8194184376	7057801526	8481490724	3962964308	8890733821	6344907299	7966779086	8918734343	8715348
K1(4) =	(-1)	0.	1248349888	7268431470	3841799808	0606848384	1584988625	8457917075	8058173093	0661421967	0303664831	173739
K1(5) =	(-2)	0.	4044613445	4521642083	6502183754	0611303019	7252633154	6144481737	4765844353	6063427608	4167173695	0531
K1(6) =	(-2)	0.	1343919717	7355090056	5154844431	4547761360	3212721831	8603325881	1582016060	5383906764	9008532934	427
K1(7) =	(-3)	0.	4541824868	8489697123	9959402710	2436312679	9741919564	2249340623	1051131250	2497023217	3463773090	77
K1(8) =	(-3)	0.	1553692118	0500113391	6862450622	4746211170	6512287261	6157079566	3294575684	6478715286	5554420450	3
K1(9) =	(-4)	0.	5363701637	9451945249	4703820199	3285734780	3413707788	5054754058	8756913790	9286315747	8613586267	
K1(10) =	(-4)	0.	1864877345	3825584596	8168581223	7167468166	6880102634	0541215150	9867316138	0240728520	322296174	
K1(11) =	(-5)	0.	6520860674	5808860555	3339496356	5892179647	4908650920	1775675597	0555470845	8886487069	000276467	
K1(12) =	(-5)	0.	2290757464	7671878159	2297207537	9650328585	0369968825	2395599015	7496657719	7575159020	05772228	
K1(13) =	(-6)	0.	8078588412	2023473318	1929581124	0226201063	5144523861	7616791808	9672004465	8054003547	3298331	
K1(14) =	(-6)	0.	2858343653	4402496632	4276643536	8002635031	5625241503	3340140117	4352481418	9847086275	0850015	
K1(15) =	(-6)	0.	1014172936	9762091810	0038968321	3643782738	9115725493	6534859657	5482355359	1404417458	08647	
K1(16) =	(-7)	0.	3607157117	5287796880	8249370805	0950212435	6829205973	8004478430	8492524795	6959211141	0239	
K1(17) =	(-7)	0.	1285704167	1666646372	8156080081	7099838184	8371474355	2223330282	9381360619	4756119181	7839	
K1(18) =	(-8)	0.	4591249627	7402409128	4758738091	2743071301	4044947031	0510309088	8884880816	6394783722	172	
K1(19) =	(-8)	0.	1642266970	3822790127	8588788929	0478525703	7882433356	8566043833	7430746465	2267621839	154	
K1(20) =	(-9)	0.	5883057969	5570381776	5028217154	2810542332	2660178347	1528291385	9416506728	5354178218	4	
K1(21) =	(-9)	0.	2110299223	3127965237	8141603217	7376127143	0805602281	6557709091	3925461770	6027087342	3	
K1(22) =	(-10)	0.	7578981163	4853310898	0376375710	1132493956	6134687719	6083783107	9038174951	632420429		
K1(23) =	(-10)	0.	2724930589	5749761133	9110663823	4846933129	9025409479	0559677021	9475837115	042269820		
K1(24) =	(-11)	0.	9806998930	4411444159	0641439183	2718053396	4084196344	7833323166	5895552314	48784800		
K1(25) =	(-11)	0.	3532778073	1999337701	9034564518	0704461567	3594741307	7772575044	9961508759	0672875		
K1(26) =	(-11)	0.	1273690779	2389236217	3462818203	3256085862	9394815563	9226819804	2271658738	430442		
K1(27) =	(-12)	0.	4595689403	6992327466	6915625534	8171575748	5165101350	2850668358	3018008867	87450		

K1(28) = (-12) 0. 1659400107 3320099344 1462298275 0378841030 0978425816 5566661041 6831880351 170
K1(29) = (-13) 0. 5995740321 2388098821 2399888670 8634305532 2708742181 0662724294 2354687537 6
K1(30) = (-13) 0. 2167732001 8915494248 6703783361 6165090175 6292256212 2074133587 50174422
K1(31) = (-14) 0. 7841899600 8340633765 5716182788 0686159208 3817835612 8221355805 32933
K1(32) = (-14) 0. 2838399271 9746694948 3539496827 9626377629 3039879613 2003144219 3
K1(33) = (-14) 0. 1027891712 2071688207 7066727704 7791852206 6539631148 5849875528 903349847
K1(34) = (-15) 0. 3724169287 0688802033 0696197417 6357215676 8603554576 8511756650 644813589
K1(35) = (-15) 0. 1349917834 0011056861 8652451760 1296181659 0704395280 6184750159 6725406
K1(36) = (-16) 0. 4895193726 8349687557 9371723585 6673824860 4649024514 8699173351 8472167
K1(37) = (-16) 0. 1775851955 1568702218 9204925550 8574229941 3493089439 0676060693 242720
K1(38) = (-17) 0. 6444785880 4236983801 4822553593 2174751441 1460913814 2873386864 55693
K1(39) = (-17) 0. 2339733402 6806734366 8937455870 1181007639 1939201863 0436838817 1419
K1(40) = (-18) 0. 8497131954 8610386507 7682324316 6963652805 4847713490 8675228963 110